PRE-CALCULUS STANDARDS

Topics 1 – 8 are appropriate for January contests

Topics 1 – 11 are appropriate for February contests

Topics 1 – 14 are appropriate for contests in March and April

For any month, only the standards as listed may be used.

1. Demonstrate an understanding of the theory of functions.

- find domains; ranges; an specific values of functions in functional notation.
- given two functions perform the algebra of functions including composition of functions.
- determine if a given function is:
    a. symmetric (with respect to the axes and/or origin.
    b. periodic
    c. monotonic
    d. bounded
    e. continuous
- identify and graph polynomial and rational functions and determine asymptotes.
- define and use parametric forms of functions and convert from parametric to Cartesian form.
- given a function; determine the inverse and state whether or not the inverse is a function.

2. **Demonstrate an understanding of connection between circular and trigonometric functions and their inverses.**

- evaluate circular and trigonometric expressions involving any of the six functions and their inverses.
- given the equation for a circular (trigonometric) function; identify and/or sketch the graph and the graph of its inverse relation and state the domain and range of the original
- function and its associated inverse function.
- identify its equation when given a graph of any of the six circular functions.
- state the period; amplitude; phase shift; and vertical shift of a circular function and/or graph of the function.

3. Demonstrate an understanding of the trigonometric identities.

- prove that a given trigonometric equation is an identity by applying the Pythagorean relation and reciprocal identities.
- prove that an appropriate trigonometric equation is an identity when given the sum and difference formulas for the cosine; sine; and tangent.
- prove that an appropriate trigonometric equation is an identity when given the double order formulas for sine; cosine; and tangent.
- prove that an appropriate trigonometric equation is an identity when given the half-angle formulas for sine; cosine; and tangent.

4. Demonstrate the ability to apply trigonometry to problem solving situations.

- solve a right triangle given two sides; or a side and an acute angle.
- use the appropriate trigonometric function(s) to solve problems involving right or oblique triangles.
- apply the Law of Sines.
- apply the Law of Cosines.
- find the area of an oblique triangle.
- estimate the solution to a problem involving a right or oblique triangle.
- in the SSA case determine whether 0; 1; or 2 triangles exist and determine the

- triangles (if they exist)

5. Demonstrate the ability to solve a variety of trigonometric (circular) equations.

- find the general solutions to a trigonometric equation
- find particular solutions to a trigonometric equation within a given domain.
- solve equations involving inverse of circular/trigonometric functions.

6. Demonstrate an understanding of conic sections and loci.

- given the description of a locus determine the equation of the locus.
- given the equation of a line determine slope and y-intercept; and graph the line.
- given the equation of a circle determine the center and radius; and graph it.
- given the equation of a parabola determine vertex; focus; and directrix; and graph it.
- given equation of an ellipse in standard form; determine the center; foci; and vertices; graph it.
- given the equation of a hyperbola in standard form; determine the foci; vertices; and asymptotes; and graph it.
- determine new equations resulting from translation or rotation of axes.
- identify the graph of any second degree equation.
- express a quadratic equation in general form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ and use $B^2 - 4AC$ to distinguish conics.
- recognize degenerate and imaginary cases.

7. Demonstrate an understanding of the relationship between exponential and logarithmic functions and their application to problem situations.

- evaluate expressions involving rational exponents.
- sketch the graphs of exponential functions and logarithmic functions of different bases.
- solve equations involving exponential functions and logarithmic functions.
- solve real-world problems involving exponential functions and logarithmic functions.
- simplify expressions using the relationships between logarithms and exponents.
- express the number e and the expression 'e to the x' as infinite series

8. Demonstrate the ability to solve problems using concepts from matrix algebra.

- apply determinants to solve systems of equations.
- invert a square matrix.

*****************END OF STANDARDS FOR JANUARY****************

9. Demonstrate the ability to solve problems using vectors.

- find a vector in standard position equal to a given vector.
- determine magnitude and direction of vectors.
- identify perpendicular and parallel vectors.
- determine the measure of the angle between two vectors.
- resolve a vector into component vectors.
- add and subtract vectors and multiply a vector by a scalar.
- find the dot product of two vectors.
- use vectors to solve real world problems.

10. Demonstrate an understanding of polynomial and rational functions; their parametric equations and their graphs.

- given a polynomial function determine intercepts and sketch the graph.

- given an equation of rational function determine intercepts and asymptotes and sketch the graph.
- given a set of parametric equations sketch the graph.

11. Demonstrate an understanding of graphs in the polar coordinate system and their relation to the Cartesian coordinate system.

- graph points in the polar coordinate system.
- convert between polar coordinates and Cartesian coordinates.
- express complex numbers in polar or trigonometric form.
- convert equations in polar form to Cartesian form.
- convert equations in Cartesian form to polar form.
- graph polar equations and identify specific types (roses; limacons; spirals; and conics)
- use de Moivre's theorem to find powers and roots of complex numbers.

*****************END OF STANDARDS FOR FEBRUARY*****************

12. Demonstrate understanding of mathematical induction and sequences and series.

- given an expression of rule for the nth term find any term of the sequence.
- given a sequence find a formula for the nth term in the sequence.
- find the nth term of a binomial expansion.
- find the sum of an arithmetic series.
- find the sum of a finite or infinite geometric series if it exists.
- define convergent and divergent sequences and series, determine limits if they exist.
- determine whether a sequence is increasing or decreasing.
- find the least upper bound and greatest lower bound of a sequence if they exist.
- express a series in sigma notation.
- use mathematical induction to prove series formulas.
- use mathematical induction to prove inequality formulas.

13. Demonstrate the ability to solve problems using probability and statistics.

- find probabilities of simple events.
- find probabilities using venn diagrams.
- find probabilities of mutually exclusive events.
- find probabilities of independent events.
- define an event and/or the complement of an event.
- find probabilities of the complement of an event.
- find conditional probabilities.
- find probabilities in binomial distributions.
- determine a standard (z) score in a normal distribution.

14. Demonstrate an understanding of the concept of limits and its applications.

- geometrically illustrate functions for which x increases without bound and find limits, if they exist.
- find when possible for any neighborhood of a number L; a neighborhood of a point *a* such that f(x) is in the neighborhood of L when x is in the neighbor hood of *a*.
- calculate limits of functions using theorems about limits.
- geometrically illustrate functions which are continuous at a point and/or continuous on an interval.
- given a rational function f(x) find the limit if it exists at a point of discontinuity.
- using the definition of the derived function of f(x) find the derive function.
- determine the equation of tangents to graphs of curves given the slope formula.

END OF STANDARDS FOR MARCH